Network Science and Models - Final Project

Communication in Mobile Geometric Graphs

Karthik K Gopalakrishnan, Igor Kadota

1 Introduction

Aerial search missions, air freight, mapping the seafloor or transporting people. Those very different scenarios have some common aspects. They are performed in a distributed manner by mobile nodes in a geographic area. Moreover, nowadays, all of them tend to employ autonomous vehicles (UAVs, AUVs or self-driving cars).

In such scenarios, communication has a central role. Nodes must communicate with others to coordinate activities. Numerous studies [1–6] have investigated communication performance of Mobile Ad Hoc Networks. In this project we propose a simple model, based on [7–9], to assess communication trade-offs in MANETs using techniques from Mobile Geometric Graphs (MGG). We use the MGG as a support to the communication process in order to analyze its performance under diverse contexts.

2 Related Work

There are two aspects in this work, mobility and communication. Surveying mobility models in the literature we find different types of models: random models, models with temporal or spatial dependency, group movement models, et. al. [10]. Here we focus on models that can emulate nodes moving independently with no spatial dependencies. Two commonly used models with that goal are:

- **Random Waypoint Model**: the most frequently used to compare routing schemes for MANETS [6, 10], it is even included in some network simulation softwares such as ns-2. It consists of nodes selecting a destination in the unit square at random and traveling to the destination with velocity $v_i \overset{d}{\sim} \mathcal{U}([0, V_{max}])$. When it reaches the destination, it waits there for a pause time (T_{pause}) and the process restarts;
- **Random Walk Model**: also known as Brownian Motion. Used to model unpredictable movement. The main differences with the Random Waypoint are: nodes do not sample destinations, but directions; and samples occur at time steps, rather then when the node reaches the destination;

When considering aerial or underwater vehicles, Random Waypoints could be used with $T_{pause} = 0$ and Random Walks with a high time step, but even in those cases, these models are likely to present steep turns (as in fig.1 left), what is not realistic for our scenario.

For that reason, in Sec.3 we propose a mobility model which extends the Random Walk Model. The basic idea is to have random movement with a preferential direction (refer to fig.1 right), what is consistent with the applications previously discussed.

Another part of this project concerns the communication process that runs on the MRGG. When we look at previous work on the literature that studies the intersection of communications and mobile nodes, we find a number of different approaches. A small sample of them are:

- [11] presents a survey of routing algorithms that use information of the node position to increase communication performance;

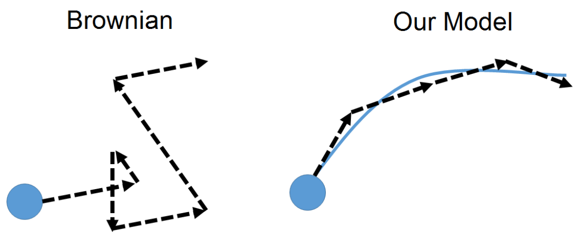


Figure 1: Mobility Types

- [6] proposes two mobility models to emulate movement of cars in a city (Freeway Mobility Model and Manhattan Mobility Model) and give insights into how mobility impacts routing performance;
- [12] proposes an algorithm that predicts the future locations of nodes to assist a Velocity-aided routing mechanism.

With previous work on mobility models and communications discussed, we introduce our mobility model in the next section.

3 MGG Model and Motivation

The proposed model is illustrated in fig.2. We consider a set of nodes $\{X_i\}_{i=1}^N$ that evolve with time, $t \in \{1, 2, \cdots\}$, according to:

$$\begin{aligned} X_i^t &= X_i^{t-1} + (\|U\| e^{j\hat{\theta}_i^{t-1}}) * \Delta t & (1) \\ \hat{\theta}_i^t &= \hat{\theta}_i^{t-1} + \Delta\hat{\theta}_i^{t-1} \quad \text{where} \quad \Delta\hat{\theta}_i^{t-1} \overset{d}{\sim} \mathcal{N}(0, \sigma^2) & (2) \end{aligned}$$

the velocity is described in polar coordinates $\vec{v_i^t} = (\|U\|, \hat{\theta}_i^t)$ and the initial parameters are $\hat{\theta}_i^0 \overset{d}{\sim} \mathcal{U}([0, 2\pi])$ and $X_i^0 \overset{d}{\sim} \mathcal{U}([0, 1]^2)$ for all nodes.

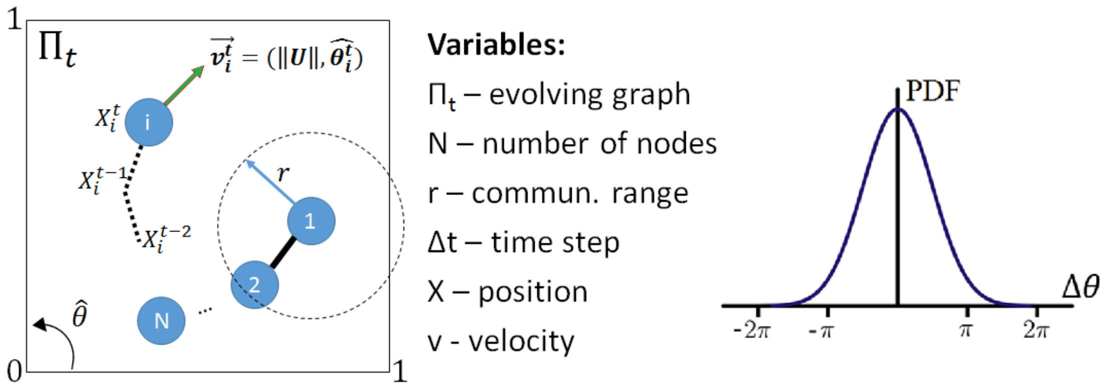


Figure 2: Model and variables

Notice that with this model, when $\sigma \to 0$, all nodes move in straight lines and bounce from the edges. On the other hand, when $\sigma \to \infty$, there is no preferential direction and nodes will move in a Brownian fashion.

The communication aspect of the model is captured by the parameter r. Nodes that are separated by less than r from one another can communicate (Unit graph model [11]). In essence, high r leads to a connected graph, while, low r means that nodes must take advantage of mobility in order to transmit messages.

4 Communication Model

In this section we discuss our communication model and routing assumptions. We simulate the MGG and the communication process with a packet having a random source-destination pair. The objective is to assess two parameters: the **number of hops** that are necessary for the packet to reach the destination and the **time** it takes for this to happen. Observe that number of hops represents resource utilization, meaning that the higher the number of hops, the more resources were utilized. On the other hand, time delay impact some specific types of traffic; real-time packets such as voice are known to have strict delay requirements. One of the goals of this project is to depict the trade-off between those parameters.

This trade-off is assessed in two different contexts. First we consider a routing independent scenario and extract from it some insights into the relation hops/time. Then we create a routing scheme based on those insights. In this section we focus on the first case only, the second is described in Sec.5.2.

For the communication to be independent of the routing scheme, we assume broadcast transmissions. This assumption is illustrated in fig.3. The basic idea is that nodes transmit the packet to all their neighbors as soon as links are created. In addition, after receiving the data, nodes never discard it, for they might need to transmit it multiple times. Notice that this broadcast scheme makes our communication process routing independent. This is because the broadcast is the union of all possible paths.

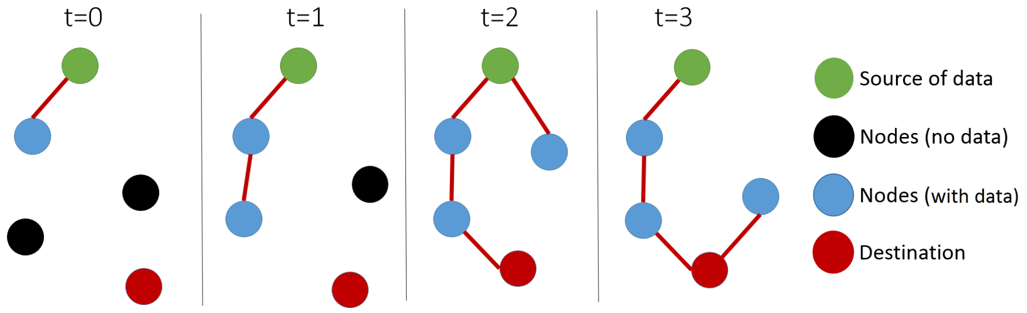


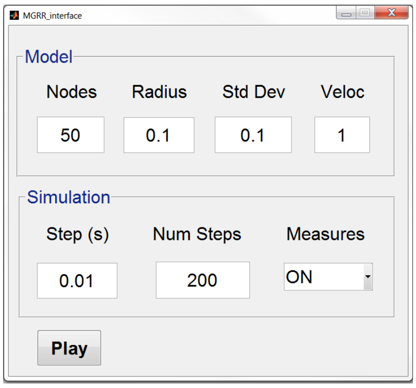
Figure 3: Broadcast Dynamic

An example of the mentioned trade-off can be extracted from the figure by observing that the destination received two copies of the same data with different delays and number of hops. The first copy was delivered at $t = 2$ with 3 hops and the second copy at $t = 3$ with 2 hops. Those important metrics (and others) will be further assessed in the next sections.

5 Simulations and Results

Based on the Mobility Model proposed in Sec.3 and the Communication Model in Sec.4 we create a custom-made simulation in the MATLAB environment to asses the impact of mobility on the communication process. Two pictures of the simulations are in figs.4 and 5. In the first, we present the interface we created to easily change the main simulation parameters, namely: Nodes (N), Radius (r), Standard Deviation (σ), Velocity ($\|U\|$), Step (Δt) and Number of Steps (which is the total number of steps in a simulation). In the second we have a frame of the run-time results.

Observe that in fig.5 we have two measures being made while the simulation is running. The graph at the top gives the fraction of nodes which have the data at a given time, and the one at the bottom keeps track of the number of hops and time delay of each packet received by



Model

Nodes	Radius	Std Dev	Veloc
50	0.1	0.1	1

Simulation

Step (s)	Num Steps	Measures
0.01	200	ON

Figure 4: Simulation Interface

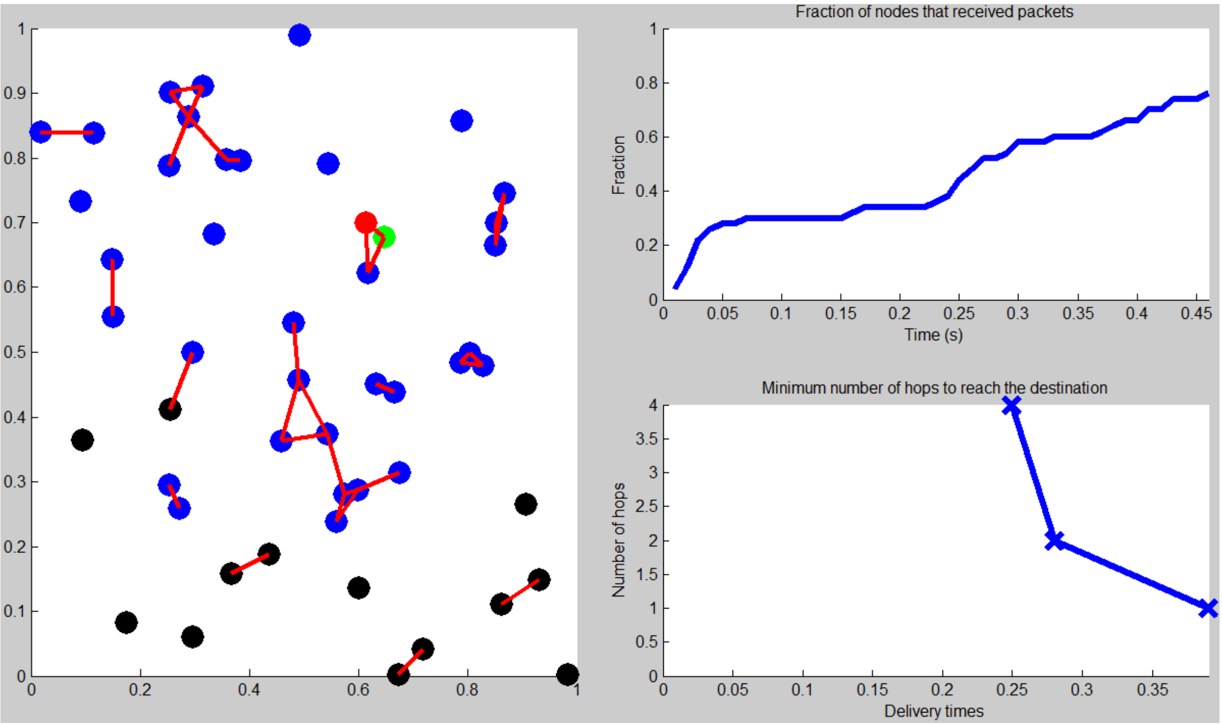


Figure 5: Simulation

the destination. With the bottom plot we can see that the first packet reached the destination at time $t = 0.25$ with 4 hops, the second at time $t = 0.27$ with 2 hops and the third at time $t = 0.39$ with 1 hop. If we carefully look at the source and destination nodes in fig.5, we can see the meaning of a 1 hop delivery. With the simulation introduced, we now discuss the main results of this project.

5.1 Results with Broadcast

In this section we present the simulation results we obtained with the Broadcast routing scheme. The first result was introduced in the previous section: fraction of nodes that have the message versus time. In fig.6 we show how this fraction behaves when we change the standard deviation of the nodes. We ran this simulation 100 times with $N = 50$ and $r = 0.1$. The results are the averages over the simulations.

Assessing this result, we conclude that the higher the standard deviation, the longer it takes

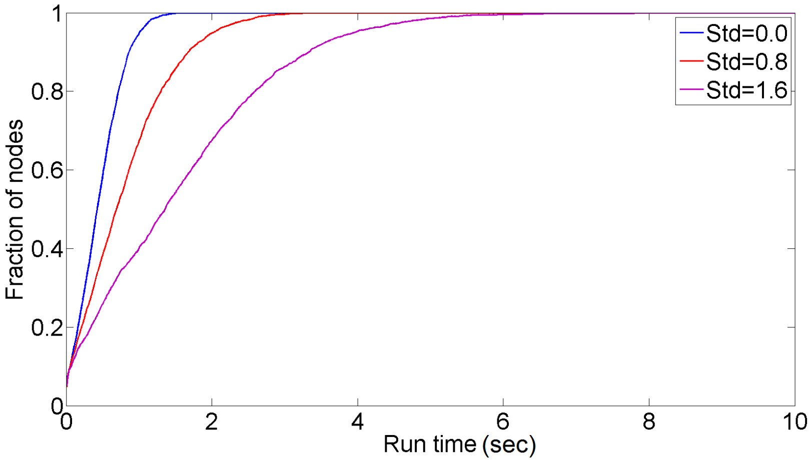


Figure 6: Fraction of nodes covered

for the packet to spread across the network. This is because a higher standard deviation implies in less preferential direction, thus, the nodes with the packet tend to stay on a limited area of the unit square. A direct implication of this result is that mobility can have a significant impact in communication. Next, we assess some extreme cases of this impact altogether with the hop/time trade-off.

Consider a node with a delay sensitive message (e.g. its position) in a fully connected graph. In this case mobility has no impact in communication. The source will send the message as fast as possible through the network until it reaches the destination. On the other extreme is a node with a message which is not delay sensitive. In this case, as the message is not urgent, if we have an advantageous velocity, we might as well aim at saving some network resources. Perhaps, if plausible, we can aim to deliver the message to the destination in 1 hop, this being an example of a communication strategy driven by mobility. Both extremes are illustrated in fig.7. The first case will be called **Min-time**, because packets need to be delivered in minimum time (using as much resources as needed), and the second **Min-hop**, meaning that we can save network resources.

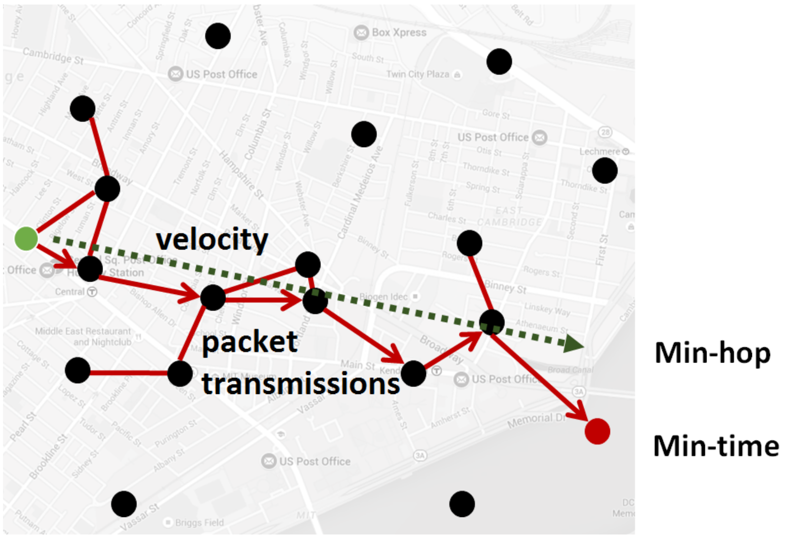


Figure 7: Minimum Time versus Minimum Hop

For assessing the case of delay sensitive messages, we keep track of the minimum time the message needs to reach the destination. We run the simulation 200 times for each combination of r and σ and a fixed $N = 50$; the averages are displayed in fig.8. From this plot, a well-known result is evident: the connectivity threshold ($r = \sqrt{log(n)/n}$). We can see that above the threshold (connected graph), as discussed, mobility has no effect on the minimum time. On the other hand, below the threshold, it is clear that mobility has a significant impact: the smaller the σ, the smaller the delivery time.

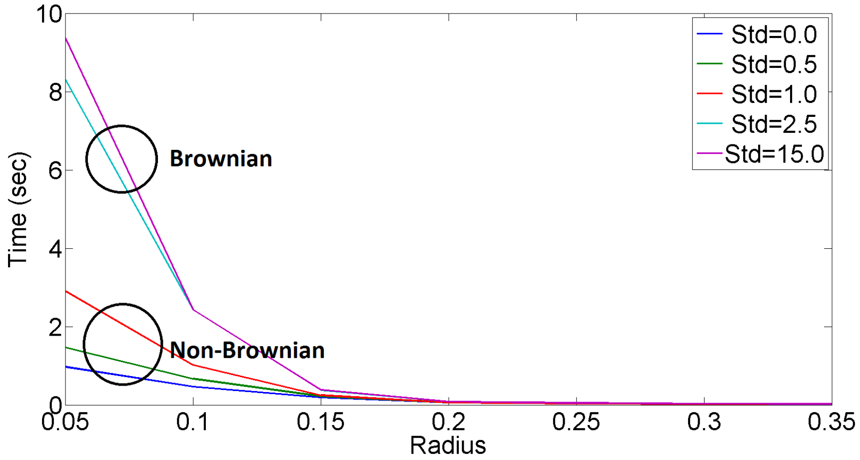


Figure 8: Minimum Time

When we analyze the Min-hop case, something interesting happens. In fig.9 we simulate a network with $N = 50$ and $r = 0.2$ and keep track of the time it takes for the message to reach the destination in exactly 1 hop (blue curve), 2 hops (red curve) and 3 hops (black curve). We can see that when we aim at saving as much resources as we can (1 hop), we get a time which is relatively high and strongly dependent on mobility. On the other hand, when we relax this constraint to a more generous scenario (2-hop) we get a better time and less dependency. Interestingly, if we choose an even more relaxed scenario (3-hop) it does not make much of a difference.

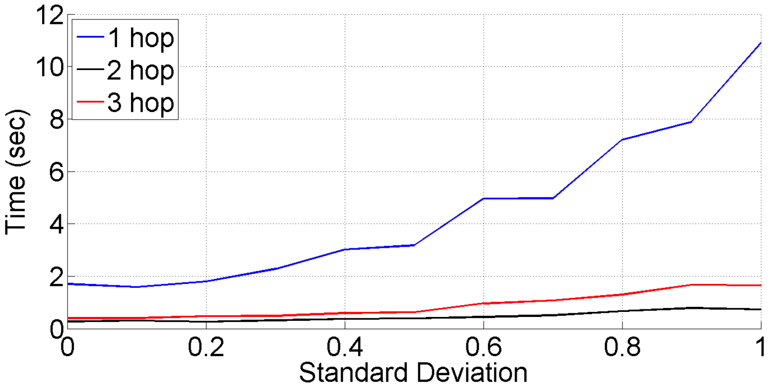


Figure 9: Minimum Hop

With this last result in mind, we try to get insight into the packet routes. As discussed, the Broadcast is the union of all possible routes, thus, we cannot know by the previous result what a good route looks like. Aiming at this type of insight we move from the Broacast case to the Unicast case in the next section.

5.2 A new velocity based routing formulation

In the previous section we have seen the performance of a broadcast communication model. Broadcast communication is resource intensive and is not a preferred strategy when there is a fixed destination node. In this section we propose a simpler greedy heuristic to navigate packets in a mobile random geometric graph.

In a Unicast communication model, information is passed from one node to another and the sender does not retain a copy. Thus there is only one node at any given time that has the message. The Unicast model has an associated message passing algorithm to decide when to transmit a message. There exists a large body of literature for routing in static ad hoc networks (for a survey, see [11]). For mobile networks, one class of routing protocols use network topology information (which is periodically updated and maintained) to optimally route data. Routes can be discovered at the time of requests or can be evaluated and stored. Position based routing is another possible strategy [13]. There is limited literature which make use of mobility models of the nodes while designing a routing scheme. A velocity aided routing scheme (VAR) proposed in [12] transmits data based on the relative velocity between the source and destination. Inspired from the VAR protocol and the Kleinberg‘s navigation model used in class, we propose a greedy algorithm for our Unicast message transmission.

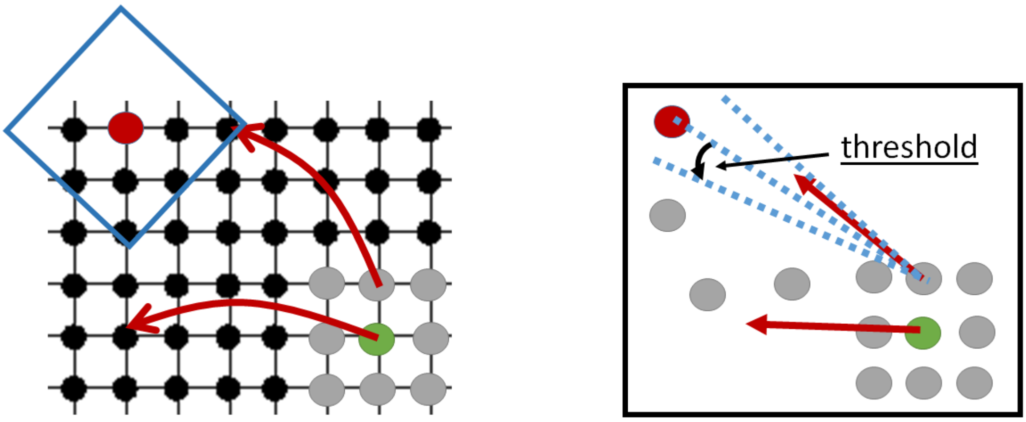


Figure 10: Kleinberg‘s lattice (Left) and velocity based routing (right)

In the Kleinberg model, a source node can find a destination by either taking all the small hops along the lattice or taking the long hops (shortcuts) to find the destination node (Figure 10). Both these aspects are considered in the new routing strategy we propose.

Notation: Let node i currently have the message/data and j be another node connected to r. Node i has a position (x_i, y_i) and velocity orientation of θ_i. Node j has position (x_j, y_j) a velocity orientation θ_j. The destination node is located at (x_d, y_d)

The heading deviation for a node is defined as the minimum change in heading angle so that the velocity is directed towards the destination (x_d, y_d). For a node i, it is denoted by $\Delta\theta_i$ For every node, define $\hat{\theta}_i$ as

$$\hat{\theta}_i = tan^{-1}\left(\frac{y_d - y_i}{x_d - x_i}\right) \qquad \text{if } x_d - x_i > 0$$

$$\hat{\theta}_i = \pi + tan^{-1}\left(\frac{y_d - y_i}{x_d - x_i}\right) \qquad \text{if } x_d - x_i < 0$$

$$\hat{\theta}_i = \pi/2 \qquad \text{if } x_d - x_i = 0 \text{ and } y_d > y_i$$

$$\hat{\theta}_i = 3\pi/2 \qquad \text{if } x_d - x_i = 0 \text{ and } y_d < y_i$$

Similar formula for node j is also used. To evaluate the heading deviation we use

$$\Delta\theta_i = min(|\hat{\theta}_i - \theta_i|, 2\pi - |\hat{\theta}_i - \theta_i|)$$

The algorithm passes the message from i (which has the data) to j if the following condition is satisfied:

$$\Delta\theta_j < \Delta\theta_i \text{ and } \Delta\theta_j < \theta_{threshold}$$

$\theta_{threshold}$ is an external parameter which tunes the greedy algorithm. If $\theta_{threshold} = \pi$, then the packet is transmitted to j whenever the velocity component of j in the direction of (x_d, y_d) is higher than that of i. This is similar to the navigation in Kleinberg's lattice where we take any path as long as the distance to the destination is decreasing. The introduction of $\theta_{threshold}$ is used to bias the greedy algorithm to look for only those paths that take the message very close to the destination. Considering the Kleinberg's lattice analogy, this is similar to looking for the shortcuts. $\theta_{threshold}$ determines the minimum heading error criteria for the data to be transmitted. Note that $\theta_{threshold} \in [0, \pi]$. The variation of the system performance with respect to $\theta_{threshold}$ is studied through simulations.

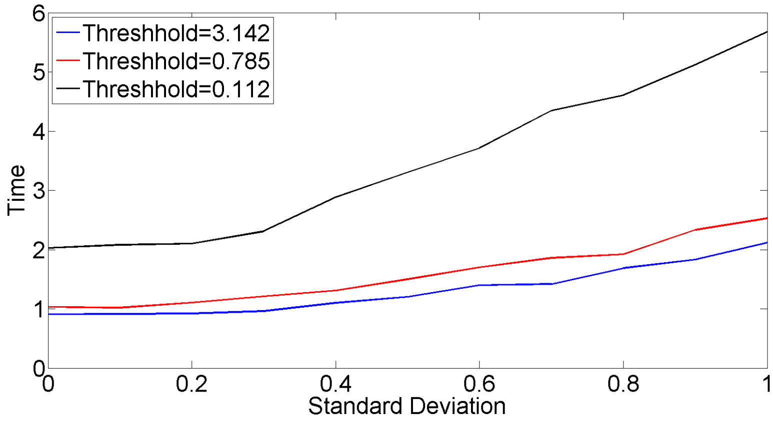


Figure 11: Time taken by greedy routing to send packet to destination for different thresholds

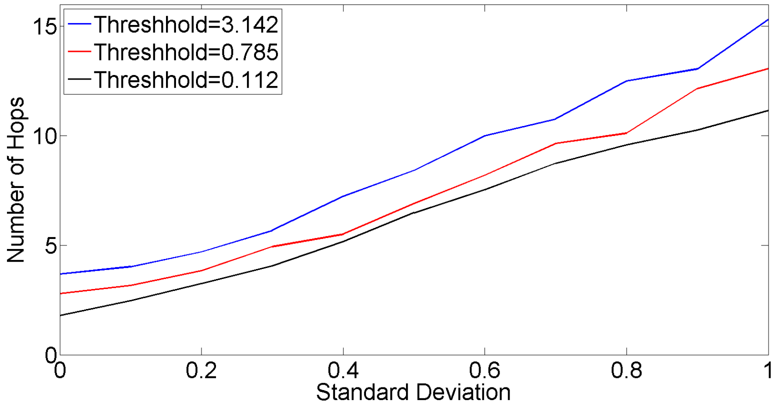


Figure 12: Hops taken by greedy routing to send packet to destination for different thresholds

Fig.11 shows the time taken for the packet delivery from a particular source-destination pair. The data is an average of 500 simulations carried out with N=50, r=0.1 and σ varied form 0 to 1. The delivery time is plotted for different values of $\theta_{threshold}$. For the same parameters, the number of hops taken for the packet to reach the destination is shown in fig.12.

As the standard deviation increases (mobility increases), the time and number of hops increases irrespective of the threshold. This is as per our expectation based on previous results. Further, we notice an interesting trade-off in fig.11 and 12. Lower thresholds (highly selective hops) leads to the low hops. But then the time taken is higher. This trade-off is better captured in fig.13, where we fix the mobility ($\sigma = 0.1$) and plot the time versus number of hops graph. Based on the application and nature of information to be transmitted, the system designer may appropriately choose the value of $\theta_{threshold}$

The comparison of the greedy algorithm with the broadcast method is described in the next section.

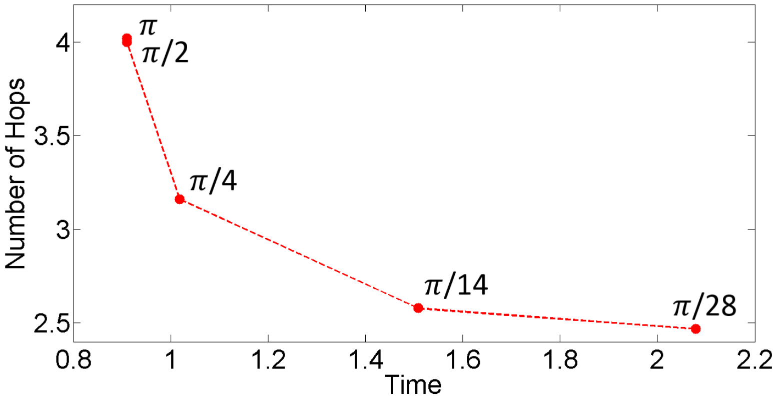


Figure 13: Trade-off between hops and time. $\sigma = 0.1$, N=50, r=0.1

5.3 Comparison of both routing strategies

We want to asses the performance of our proposed greedy heuristic with a broadcast model. For this comparison, we consider the two aspects: minimum time and minimum hops. The results of 500 simulations carried out with N=50, r=0.1, $\sigma = 0.1$ is summarized in Table 1 and Table 2.

Metric	**Broadcast**	**Greedy** ($\theta_{threshold} = \pi/28$)
Delivery time	0.9	2.1
No. of hops	2	2.5

Table 1: Comparison of minimum hops

Metric	**Broadcast**	**Greedy** ($\theta_{threshold} = \pi$)
Delivery time	0.7	0.9
No. of hops	2.8	4.0

Table 2: Comparison of minimum time

For the minimum time, we know that the broadcast routing will always have the best performance. The greedy navigation obtains best min-time performance when $\theta_{threshold}$ is high. Therefore it is initialized with $\theta_{threshold} = \pi$. The number of hops however is not considered and the greedy model takes 4 hops on an average compared to 2.8 by the broadcast method. On a positive note , the time taken by the greedy model is close to the broadcast. This means that the 'route independence' that the broadcast model provides is not a significant benefit when the node mobility is high.

Consider the case of minimum hops. In the broadcast model, the minimum number of hops required will asymptotically tend to 1 as T tends to infinity. In previous results we note that the time taken for 2 or more hops is significantly lower across a range of mobility values. It is reasonable to assume that in a practical implementation, a 1-hop constraint will not be imposed. So we compare the greedy routing model with a 2-hop broadcast. Here, the $\theta_{threshold} = \pi/28$, a small value so that the routing strategy selectively chooses the shortcuts and minimizes hops. The number of hops is 2 for broadcast and 2.5 for our greedy algorithm. As expected, the time for delivery is much higher for the greedy heuristic.

6 Summary

In this report, we study the communication aspects involved in mobile random geometric graphs. We list out the main conclusions of our study.

- Existing models like the random waypoint, random walk or Markov models do not explicitly account for mobility or preferential directions. Although preferential motion can be introduced in a Markov setting, we propose a much simpler mobility model. A Gaussian direction distribution with the current heading as mean is used. The variance of the Gaussian distribution is a measure of the mobility of nodes. This allows us to study the entire spectrum of mobility from direct straight line motion to Brownian motion.

- Below the connectivity threshold for the graph, mobility plays a strong role in packet transmission. Higher the preferential direction, lesser is the time taken to transmit information. Above the connectivity threshold, mobility does not matter as there will exist at least one path from any source to destination.

- Simulations show that 2-hops is a good threshold for minimizing time for delivery. There is little marginal benefit in terms of delivery time when more than 2-hops are used. From a design perspective, resources need not be spent to increase the number of hops as there is very little reduction in delivery time.

- A velocity based greedy routing algorithm was introduced. The algorithm can be tuned so as to minimize hops or time and this trade-off is studied. For both the cases, the performance of the greedy algorithm was comparable to a broadcast communication. Thus, the navigation algorithm is able to send packets to the destination and find a good approximate route by using only local information.

References

[1] M. J. Neely, E. Modiano, and C.-P. Li, "Fairness and optimal stochastic control for heterogeneous networks," in *Proceedings of IEEE INFOCOM*, Mar 2005.

[2] I.-H. Hou, "Providing end-to-end delay guarantees for multi-hop wireless sensor networks," in *Globecom 2013*, 2013.

[3] G. R. Gupta and N. B. Shroff, "Delay analysis and optimality of scheduling policies for multihop wireless networks," *IEEE/ACM Transactions on Networking*, 2011.

[4] R. Li and A. Eryilmaz, "Scheduling for end-to-end deadline-constrained traffic with reliability requirements in multihop networks," *IEEE/ACM Transactions on Networking*, 2012.

[5] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control*, 1992.

[6] F. Bai, N. Sadagopan, and A. Helmy, "Important: a framework to systematically analyze the impact of mobility on performance of routing protocols for adhoc networks," in *Proceedings of IEEE INFOCOM*, 2003.

[7] B. Bollobas, R. Kozma, and D. Miklos, *Handbook of Large-Scale Random Networks.* Springer, 2000.

[8] M. Penrose, *Random Geometric Graphs.* Oxford University Press, 2003.

[9] M. Newman, *Networks: An Introduction.* Oxford University Press, 2010.

[10] T. Camp, J. Boleng, and V. Davies, "A survey of mobility models for ad hoc network research," *Wireless Communications and Mobile Computing*, vol. 2, pp. 483–502, 2002.

[11] S. Giordano, I. Stojmenovic, and L. Blazevic, "Position based routing algorithms for ad hoc networks: A taxonomy," *Ad Hoc Wireless Networking*, pp. 103–136, 2001.

[12] K.-T. Feng, , C.-H. Hsu, and T.-E. Lu, "Velocity-assisted predictive mobility and location-aware routing protocols for mobile ad hoc networks," in *IEEE Transactions on Vehicular Technology*, 2008.

[13] M. Mauve and J. Widmer, "A survey on position based routing in mobile ad hoc networks," *IEEE Network*, 2001.